PHYS1110D – Engineering Physics: Mechanics and Thermodynamics

Tutorial Problems for Week 7: Energy and Linear Momentum

**Problem 5\* – Why ?**

*(This puzzle is taken from* [*https://youtu.be/HEfHFsfGXjs*](https://youtu.be/HEfHFsfGXjs) *by 3Blue1Brown. It may give you a nice geometrical interpretation of the process of elastic collisions in one dimension)*

A block with mass starts with velocity towards another block with mass . (We set velocity to be positive if it points towards the left.) Suppose there is a wall to the left of block 2. After collision with block 1, block 2 will move towards left, and bounce back from the wall, so it may collide with block 1 again. Therefore, there will be many collisions between the two blocks, as well as collisions between block 2 and the wall.

Suppose that all collisions are elastic. The interesting thing is: for (), the total number of collisions is just the integer part of ! For example, if (i.e. ), then the number of collisions is 3 (*can you see this immediately?*); if , then the number is 314. Can you show why?

*Hint*: A clever way to tackle this problem can be outlined as follows. Whenever the number shows up, there must be a hidden circle in the problem. In fact, by defining

We can rewrite the equation of energy conservation

as that of a *circle* in the -plane:

And the equation of momentum conservation

Can be written as

It represents a *straight line*. Now think: how can we find the velocities of the two blocks after each collision by drawing in the -plane?

You may need the approximation: for (much smaller than 1 but no less than 0)

**Solution:**

<https://youtu.be/jsYwFizhncE>

*Remark:* We mention that there is another solution which requires less thinking, but a knowledge of linear algebra (in particular, *diagonalization* of matrices) and complex numbers. You can return to this problem when you are older.